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TURBULENT NATURAL CONVECTION IN A HORIZONTAL WATER LAYER HEATED FROM BELOW

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Abstract—This paper presents results of an experimental investigation into heat transfer in a water layer confined by two horizontal plates, heated from below and cooled from above. The experiment covered a range of Rayleigh numbers up to 4×10^9. The heat-transfer resistance is virtually controlled by the thermal boundary layers near the heating and cooling plates. The boundary layer releases buoyant plumes according to its autonomous mechanism. The central region of the fluid layer is almost isothermal. From visual observation and also from theoretical considerations, it was concluded that there exist convective motions having a scale comparable to the distance between the two plates and they make a dominant contribution to heat transfer in the central region.

NOMENCLATURE

c_p, specific heat;
D, distance between top and bottom surfaces;
g, acceleration of gravity;
h, heat-transfer coefficient;
l_e, size of energy-containing eddies;
Nu, Nusselt number;
Pr, Prandtl number;
q, heat flux;
Ra, Rayleigh number;
Re_l, Reynolds number, $u'l_e/\nu$;
Re_λ, Reynolds number, $u'\lambda_g/\nu$;
T, instantaneous temperature;
T_m, mean temperature, $(T_1 + T_2)/2$;
T_1, bottom surface temperature;
T_2, top surface temperature;
ΔT, temperature difference, $T_1 - T_2$;
$\bar{T}$, time-averaged temperature;
t^*, theoretical period of plume discharges;
u', r.m.s. velocity component;
V, absolute turbulence velocity;
z, distance from the bottom surface.

Greek symbols
β, expansion coefficient;
δ, thickness of thermal boundary layer, defined in equation (2);
δ_c, thickness of conduction boundary layer;
η, Kolmogorov microscale;
κ, thermal diffusivity;
λ, thermal conductivity;
λ_g, Taylor microscale;
ν, kinematic viscosity;
$\bar{\theta}$, dimensionless time-averaged temperature, $2(\bar{T} - T_m)/\Delta T$;
θ'_+, dimensionless positive temperature deviation;
θ'_-, dimensionless negative temperature deviation;
ρ, density;
τ, experimental period of plume discharges.

Subscript
m, refers to T_m.

INTRODUCTION

THE PROBLEM of natural convection in a fluid layer confined by two horizontal plates, heated from below and cooled from above, has for many years been of considerable interest [1]. While this basic kind of buoyancy induced convection seems to have incessantly been providing pure scientists with an intellectual stimulation, the knowledge of this type of convective heat and mass transfer has recently been required not only in the field of environmental science but also in various technological areas, including among others nuclear reactor design and safety. For instance, liquid sodium which is used as a coolant in a FBR-type nuclear power reactor is transferred by natural convection in argon gas layer charged to cover the free surface of liquid sodium, to the cold upper wall surfaces of various containers and pumps in the plant. The deposited solid sodium will sometimes cause trouble. The assessment of post-accident heat removal from molten fuel or molten steel is another example.

It is well known [1] that for Rayleigh numbers less than a critical value of about 1700 fluid layer is stagnant and the Nusselt number is unity. At Rayleigh numbers slightly greater than critical, steady laminar convection in the form of cells and rolls occurs. Subsequent increase in the Rayleigh number causes the flow to become unsteady and complex. It is generally accepted that the flow is fully turbulent for Rayleigh numbers greater than about 10^5 for moderate Prandtl numbers. Here we may confine our attention to a range of the Rayleigh numbers which are higher than transitional from laminar to turbulent convection.

Kraichnan [2] extended the mixing-length theory to the turbulent thermal convection, giving the following relations between the Nusselt number Nu and the Rayleigh number $Ra : Nu \propto Ra^{1/3}$ for a moderate turbulent region and $Nu \propto [Ra/(\ln Ra)^3]^{1/2}$ for extremely large Ra of the order of 10^{24}. Similar phenomenological heat-transport models have been developed by Long [3], and also by Cheung [4] for the case of fluid layer with uniform volumetric energy sources. A novel theoretical approach toward the understanding of turbulent flows which is now referred to as the optimum theory of turbulence [5] was originated concerning the thermal convection by Malkus [6]. His suggestion that turbulence arranges itself so as to maximize the heat flux between the plates provided the stimulation for the development of the optimum theory by Howard [7] and the subsequent extension of the theory by Busse [8]. The upper bound of Nu obtained by Howard was $(Ra/248)^{3/8}$, whereas Busse has shown that $Nu \leqq (Ra/1035)^{1/2}$ in the limit of infinitely high Ra. These predicted upper bounds are higher than the experimental Nusselt numbers by a factor of about three. A high speed computer technique has made it possible for a numerical investigation to be executed up to the Rayleigh number of 10^6 by Herring [9], Deardorff [10] and Elder [11]. The calculated Nusselt numbers were in considerably good agreement with experimental ones, having the dependence of the form $Ra^{1/3}$.

Experimental measurements of the heat-transfer coefficient have been performed by a number of investigators [12–17] using several kinds of test fluids of different Prandtl numbers, in a range of the Rayleigh numbers up to about 10^{10}. In the data reduction, a general form $Nu = kRa^m$ has been assumed. While the value of the exponent m was found to be near 1/3, accurate measurements gave values slightly smaller than 1/3 [14, 16]. At the same time, accurate measurements revealed discrete transitions of the heat-transfer coefficient at several distinct Rayleigh numbers [14–16, 18], the largest of which was as high as 4×10^8 [15]. Existence of these transitions had been conjectured by Malkus [6]. Vertical distributions of horizontally averaged or time-averaged physical quantities such as temperature and fluctuations of temperature and velocity were measured in several works [13–15, 19] including elaborate ones performed by Deardorff and Willis [20] and Fitzjarrald [17]. An interesting finding from elaborately measured mean temperature profiles was an existence of reversal of the temperature gradient in the almost isothermal region in the midpart of the fluid layer [13, 14]. Chu and Goldstein [14] reported that the gradient reversal was observed up to the Rayleigh number of 5×10^5. Deardorff and Willis [20] and Fitzjarrald [17], from spectral analyses of their data obtained for the Rayleigh numbers up to 1.7×10^7, insisted that in the central region of the fluid layer most part of heat is carried by convective motions having a scale comparable to the depth of the fluid layer. Very recently Simpkins and Dudderar [21] applied a technique of laser speckle photography which allowed an instantaneous two-dimensional full-field velocity distribution to be obtained solely from a selected plane, to the case of the thermal convection at a Rayleigh number of about 10^5.

The purpose of the present investigation is to clarify the mechanism of heat transfer between the two plates, paying special attention to the structure of turbulent motions induced in the central part of the fluid layer, especially at high Rayleigh numbers up to 4×10^9, the greatest possible value in a laboratory experiment.

EXPERIMENTAL APPARATUS AND PROCEDURE

A schematic diagram of the experimental apparatus is shown in Fig. 1. Water was used as a test fluid so as to realize high Rayleigh numbers. The upper and lower surfaces of the convection chamber were both made of copper plates of 5 mm in thickness and their effective heat-transfer areas were 70 cm by 70 cm wide. Side walls were made of transparent acrylic resin plates. Three sets of side walls, 5, 10 and 20 cm in height, were employed. Either of the upper and lower plates was cooled or heated by passing cold or warm water through channels fitted on the backside of the plate. The lower surface temperature T_1 as well as the upper surface temperature T_2 was measured by thermocouples embedded in the plate. Heat flux q was determined from the flow rate and temperature drop of warm water through the heating channels. Care was taken so that this temperature drop should be sufficiently smaller than the temperature difference between the upper and lower plates. As compared with the total heat flow from the bottom to top plates, heat loss from the side walls was estimated to be negligibly small. For measuring fluid temperature within the convection chamber, a chromel–alumel thermocouple of 50 μm diameter and 8 mm span was used after examining the effect of thermal conduction along the thermocouple wire [22]. The time constant of the thermocouple was estimated at 20 ms at a water velocity of 5 mm s^{-1}. The thermocouple was inserted into the chamber through the upper plate and was capable of traversing vertically the lower half of the fluid layer at the center of the chamber. Various visualization techniques were introduced to determine flow patterns as well as flow velocities. Hydrogen bubbles generated pulsatively from 10 μm diameter tungsten wire submerged in the water layer as well as sawdust dispersed in the water layer were employed as a tracer to visualize fluid motions chiefly in the central part of the fluid layer. In order to examine the fluid motions near the surface, we used an electrochemical technique making use of phenolphthalein, i.e. a pH indicator. The positive electrode was a 0.3 mm wire situated 1 cm aside from the lower surface and the lower surface itself served as a negative electrode. In this case the fluid next to the lower surface near the positive electrode wire acquired red color.

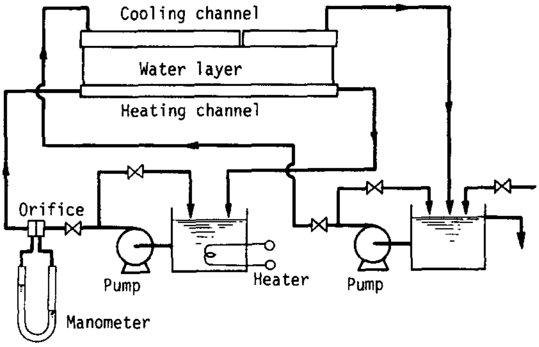

FIG. 1. Schematic diagram of the experimental apparatus.

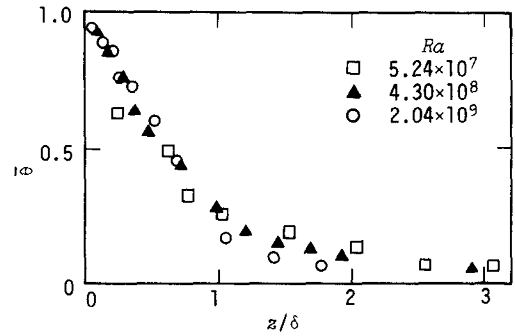

FIG. 3. Vertical distributions of time-averaged fluid temperature near the lower surface.

RESULTS AND DISCUSSIONS

Experimental results of the Nusselt number against the Rayleigh number are shown in Fig. 2. Here the Rayleigh number is defined as $Ra = D^3 g\beta_m \Delta T/(\nu_m \kappa_m)$, where D is the distance between the upper and lower plates, ΔT is the temperature difference between the two plates $T_1 - T_2$, g is the acceleration of gravity, and β, ν and κ are the expansion coefficient, the kinematic viscosity and the thermal diffusivity of the fluid, respectively. The subscript m refers to the physical property that corresponds to a mean temperature $T_m = (T_1 + T_2)/2$. The Nusselt number is defined as $Nu = hD/\lambda_m$, where h is the heat-transfer coefficient between the two plates, $q/\Delta T$, and λ is the thermal conductivity of the fluid. A linear regression of $\ln(Nu)$ on $\ln(Ra)$ through the experimental data points gives

$$Nu = 0.145\,Ra^{0.29}. \tag{1}$$

Globe and Dropkin [12] performed heat-transfer measurements using several fluids with a large variety of Prandtl numbers and have shown that the effect of Prandtl number is measurable but small. Equation (1) is in good agreement with the accumulated results of previous experiments [12–17], with the exponent of Ra being 0.29, a value near 1/3 but distinguishably smaller than 1/3.

Thermal boundary layer

Figure 3 shows vertical distributions of time-averaged fluid temperature $\bar{T}$ near the lower surface. The ordinate signifies $\bar{\theta} = 2(\bar{T} - T_m)/\Delta T$, while the abscissa is the distance from the lower surface, z, normalized by the following fluid layer thickness that gives the same heat-transfer conductance as the measured heat-transfer coefficient.

$$\delta = \lambda/(2h) = D/(2Nu). \tag{2}$$

It is easily understood that in this plotting the temperature gradient near the surface becomes -1.0. The fluid temperature approaches gradually to T_m as z increases, and fluid in the central region except 3δ from the surface is almost at a uniform temperature of T_m. Thus, the heat-transfer coefficient h of this system is virtually determined by the thermal boundary layers which develop near both surfaces. The fact that the exponent of Ra in expression (1) is near 1/3 means that h is almost independent of the distance D between the two plates. This suggests that the thermal boundary layer has its own physical mechanism almost independent of the existence of the opposite surface.

Figure 4 shows variations of instantaneous fluid temperature T with time at two particular positions: $z/\delta = 0.53$ and 3.53. As is typified by Fig. 4, a temperature trace at a position $z/\delta > 1$ has large

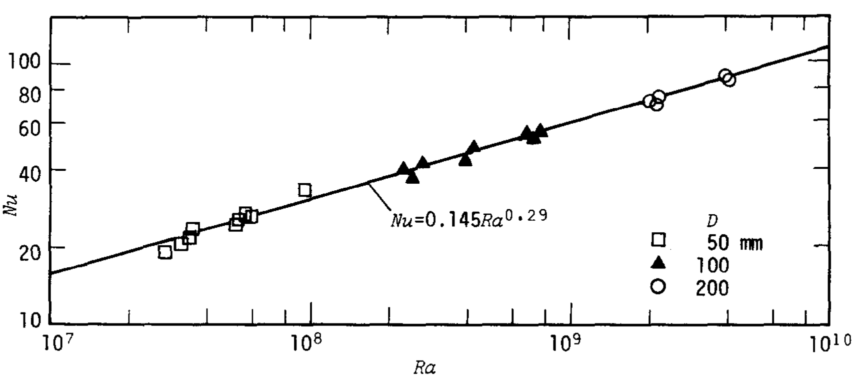

FIG. 2. Experimental results of Nusselt number against Rayleigh number.

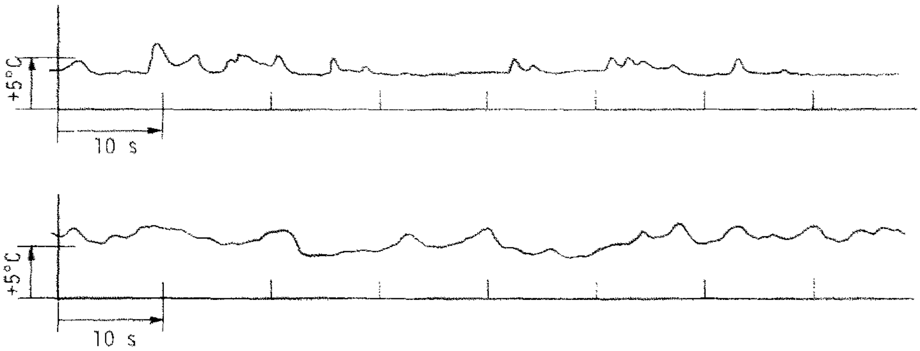

FIG. 4. Variations of instantaneous fluid temperature with time; D = 20 cm, ΔT = 14.8°C, Ra = 2.04 × 10^9; lower half: z/δ = 0.53, upper half: z/δ = 3.53.

skewness as compared with one at $z/\delta \leqq 1$. Namely, fluid temperature at $z/\delta > 1$ traces mostly a certain base value and occasionally impulsive deviations towards higher temperatures appear. These impulsive deviations are considered to correspond to discharges of warm fluid from the thermal boundary layer. By applying the electrochemical visualization technique mentioned previously, we could observe buoyant plumes in the form of sheets rising from the lower surface. Figure 5 shows nondimensionalized vertical distributions of positive and negative temperature deviations, $\theta'_+ = 2T'_+/\Delta T$ and $\theta'_- = 2T'_-/\Delta T$ where T'_+ denotes an average of several maximums appearing in positive deviation of instantaneous temperature from the time-average $(T - \bar{T})$ and T'_- is similarly defined as for negative deviation $-(T - \bar{T})$. The experimental data in Fig. 5 are somewhat scattered but agree well together irrespective of the Rayleigh number. Both θ'_+ and θ'_- increase first as z/δ goes up to 1.0, at which the total variation $(\theta'_+ + \theta'_-)$ amounts to as large as 0.6. This violent temperature variation may be attributed to the repetition of discharge of warm fluid from the boundary layer and subsequent replacement by cold fluid. As z/δ goes beyond 1.0, θ'_- decreases rapidly. On the other hand, θ'_+ keeps its high level up to about z/δ = 6, and then it turns to decrease. At z/δ = 16, almost all traces of ascending warm masses disappear. Then, from equation (2), the uppermost Nusselt number at which buoyant plumes from one surface can arrive at the opposite surface is estimated as Nu = 8. Using equation (1) this corresponds to $Ra = 1 \times 10^6$. While the foregoing descriptions may be restricted by the Prandtl number, it is noteworthy that Chu and Goldstein [14] reported from an experiment using the same water as the present investigation that when $Ra < 8 \times 10^5$ buoyant masses reach the opposite surface and coalesce to form large masses, resulting in a temperature-gradient reversal.

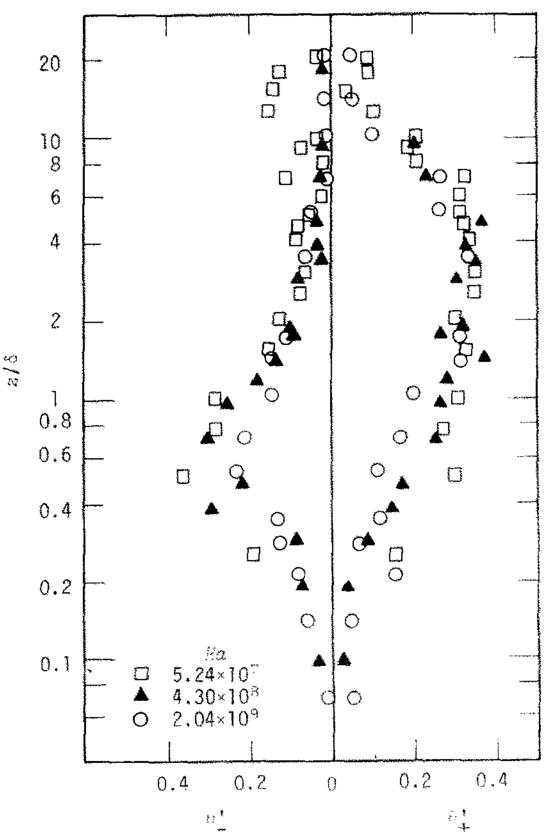

FIG. 5. Vertical distributions of positive and negative temperature deviations.

Howard [23] has proposed the following simple model for the periodic release of buoyant plumes from the boundary layer. At time t = 0, the layer of fluid is at uniform temperature T_m and the lower plate is brought to temperature $(T_m + \Delta T/2)$. Initially a conduction boundary layer having thickness $\delta_c = \sqrt{(\pi \kappa t)}$ develops near the surface. At time t^* when a Rayleigh number defined as $Ra_\delta = \delta_c^3 g\beta\Delta T/(\nu\kappa)$ comes up to a value of 1000, the conduction boundary layer is swept up in releasing buoyant plumes. Then the process is repeated from the beginning. The period of the cyclic discharges of plumes is easily determined as $t^* = 100(\pi\kappa)^{-1}\{\nu\kappa/(g\beta\Delta T)\}^{2/3}$. The experimental mean period of plume discharges, τ, was calculated by counting the number of peaks appearing in the records of

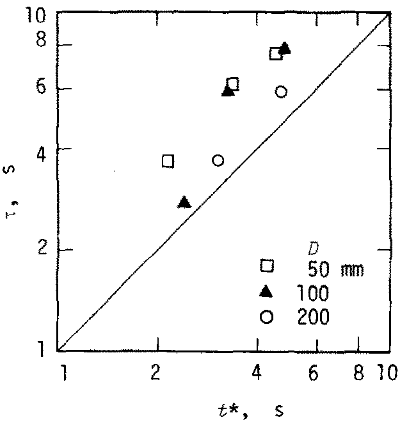

FIG. 6. Comparison between theoretical and experimental periods of plume discharges.

instantaneous fluid temperature variation such as shown in Fig. 4. Temperature records at $z/\delta = 3$ were used. From Fig. 5, θ'_+ amounts to 0.3 at $z/\delta = 3$ while θ'_- almost vanishes at $z/\delta = 3$. Therefore a sharp pulse was marked by each individual plume at this position. We counted temperature peaks whose normalized heights, $2(T - \bar{T})/\Delta T$, exceeded $0.5\theta'_+ = 0.15$. The measured periods τ are plotted against the theoretical ones t^* in Fig. 6. Since the experimental ranges of ΔT and T_m were fairly limited in the present investigation, t^* is restricted in a narrow range. It seems, however, that the foregoing simple model by Howard explains the physical phenomenon fairly successfully.

Turbulence in the central region

Figure 7 shows two samples of long exposure photographs of the flow in the vertical midplane of the

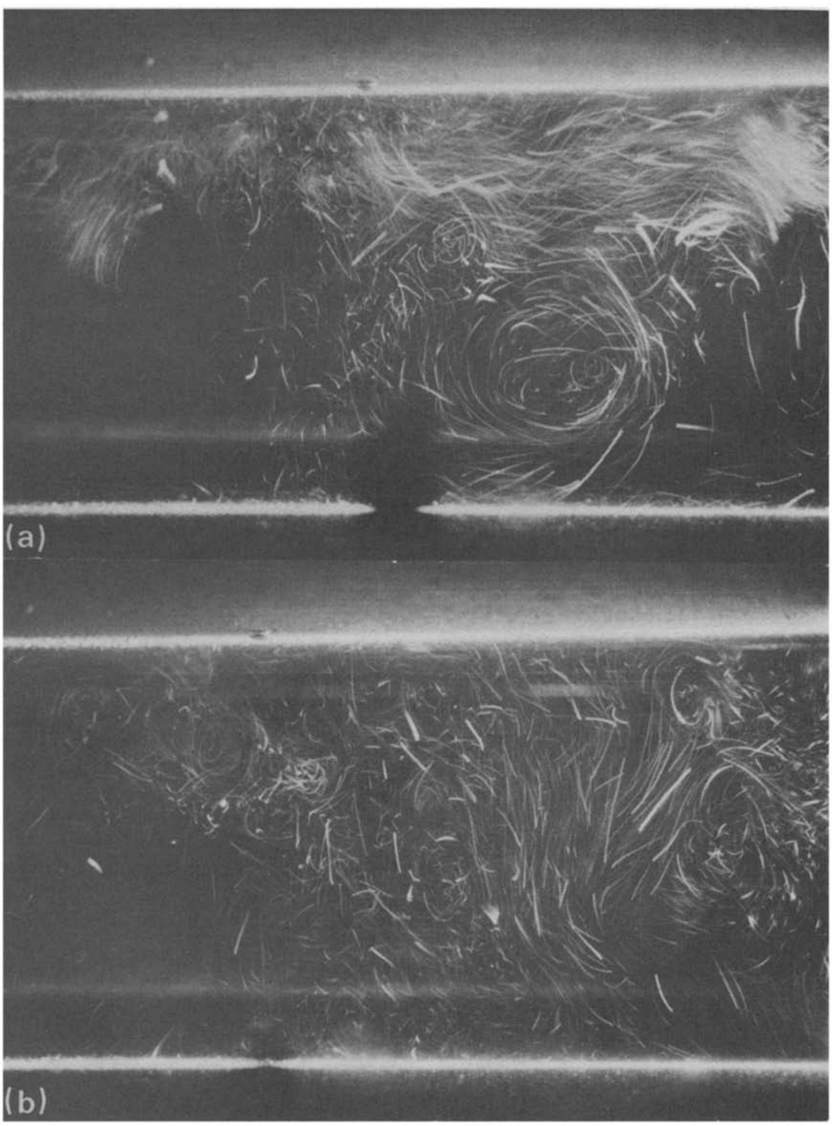

FIG. 7. Sample photographs of the flow in the vertical midplane visualized by sawdust particles; $D = 20$ cm, $Ra = 1.73 \times 10^9$; exposure time: (a) 4 s, (b) 3 s.

convection chamber visualized by sawdust particles which were in suspension in water and were illuminated by light through a slit. The light was scattered by the boundary plates, yielding upper and lower bright lines in the pictures. While eddies of various sizes are visualized in Fig. 7, a large vertical convective motion extending over the depth of the fluid layer is seen in picture (b). In the case of the smallest ratio of width to height of the chamber, 70 cm/20 cm, pattern of the large convective motion seemed to be affected to a certain extent by the existence of the side walls. Seemingly circular motions having large sizes comparable to the distance between the plates, however, were always observed in the range of Rayleigh numbers of the present experiment, although the motions neither had regular configurations such as cells or rolls nor were stationary. Namely, the site of large upward flush swayed, the fluid layer sometimes became somewhat stagnant and then the motion with different pattern from before began.

Turbulent velocity V in the central region of the fluid layer was calculated by taking an average of lengths of several long traces of sawdust particles in such a photograph as Fig. 7. Turbulent velocities were more generally measured by applying the usual hydrogen bubble technique. In this method rising velocity of the hydrogen bubble due to its own buoyancy was not negligible since V was as small as 10–20 mm s^{-1}. Then, we placed a tungsten wire horizontally in the midst of the convection chamber and adopted as V an average of several large downward velocities compensated by the rising velocity of the hydrogen bubbles. The measured turbulent velocities were made dimensionless by using the velocity scale due to Kraichnan [2] and plotted against Ra in Fig. 8. The experimental data were well correlated by

$$V/(Pr^{1/3}\kappa/D) = 1.05Ra^{0.43} \tag{3}$$

where Pr is the Prandtl number of the fluid. Simple estimation of the r.m.s. velocity component u' from the relation $u'^2 = V^2/3$ reduces equation (3) to

$$u'/(Pr^{1/3}\kappa/D) = 0.6Ra^{0.43}. \tag{4}$$

Garon and Goldstein [15] measured time-averaged r.m.s. value of the midplane vertical velocity in a water layer by applying laser-Doppler technique in a range of Rayleigh numbers between 1.3×10^7 and 2.5×10^9. Fitzjarrald [17], with a convection chamber of air designed to allow measurements to be taken along a horizontal path midway between the top and bottom plates, measured spatially averaged r.m.s. values of vertical velocity and longitudinal horizontal velocity at the midplane, in a range of Rayleigh numbers from 4×10^4 to 1.7×10^7. In Fitzjarrald's experimental range, the midplane r.m.s. value of vertical velocity is larger than that of longitudinal horizontal velocity by about 20%. These previous experimental values, however, are also correlated generally well by equation (4).

In the central region of the fluid layer, almost all heat is transferred by turbulent motions and contribution of molecular conduction is negligible. Heat transfer by turbulence involves temperature fluctuations, i.e. density fluctuations, which cause fluctuating body forces. It can be shown that the fluctuating body forces involved by a vertically upward heat flux q perform work at a mean rate $g\beta q/(\rho c_p)$ per unit mass, where ρ and c_p are the density and specific heat of the fluid, respectively [24]. This buoyant production of turbulence energy is after all consumed by the viscous dissipation by the turbulent motions in the central region. Here we may apply the isotropic turbulence theory for the purpose of a first approximation, and estimate the viscous dissipation rate as u'^3/l_e, where l_e is the average size of the energy-containing eddies [25]. Thus we have the following equation as an average over the whole fluid layer.

$$g\beta q/(\rho c_p) = u'^3/l_e. \tag{5}$$

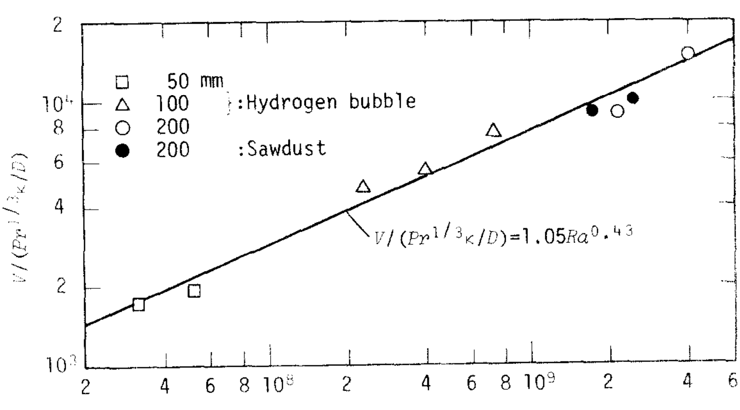

FIG. 8. Correlation of measured turbulent velocity with Rayleigh number

Substitution of the empirical equations (1) and (4) into this equation with the aid of a relation $q = Nu(\lambda/D)\Delta T$ and the definition of the Rayleigh number results in the following estimation of the size of the energy-containing eddies, l_e.

$$l_e/D = 1.5. \tag{6}$$

It seems somewhat accidental that the exponent of Rayleigh number has been completely cancelled. Physically important implication of equations (5) and (6), however, is that, in so far as the velocity fluctuations having an order of actual measurements do exist, the energy-containing eddies should at least have a size comparable to the distance between the plates, D. Otherwise, if l_e is smaller than D, the viscous dissipation term surpasses the energy production. Equation (6) is supported by the visual observations mentioned previously. It seems also compatible with the experimental results by Deardorff and Willis [20] and Fitzjarrald [17] who measured cospectra between vertical velocity and temperature from hot-wire traverses along a horizontal path and found that most part of heat flux is carried by waves having sizes comparable to or greater than the depth of the fluid layer.

Using equations (4) and (6) turbulence Reynolds number $Re_l = u'l_e/\nu$ can be expressed in terms of the Rayleigh number as

$$Re_l = 0.9Ra^{0.43}Pr^{-2/3}. \tag{7}$$

Further, by applying the isotropic turbulence theory [25], microscale Reynolds number $Re_\lambda = u'\lambda_g/\nu$ in which λ_g is Taylor microscale can be found from the relation

$$Re_\lambda^2 = 15Re_l.$$

Taylor microscale λ_g and Kolmogorov microscale η can be estimated from the relations

$$\lambda_g/l_e = 15Re_\lambda^{-1} \quad \text{and} \quad \eta/l_e = 15^{3/4}Re_\lambda^{-3/2}.$$

For instance, when $Ra = 2 \times 10^9$ and $D = 20$ cm, the foregoing various parameters are estimated as follows:

$$Re_l = 2.7\times 10^3, \quad Re_\lambda = 2.0\times 10^2, \quad l_e = 300\ \text{mm},$$

$$\lambda_g = 22\ \text{mm}, \quad \text{and} \quad \eta = 0.8\ \text{mm}.$$

Under the same conditions, the Nusselt number and the thermal boundary layer thickness defined by equation (2) are

$$Nu = 72 \quad \text{and} \quad \delta = 1.4\ \text{mm}.$$

If we assume from the beginning that $l_e = D$, the balance equation of turbulence energy (5) is transformed to

$$u' = (\kappa/D)(PrRaNu)^{1/3}. \tag{8}$$

The expression on the right-hand side of this equation was first proposed by Deardorff [26] as an appropriate velocity scale concerning the convection in the unstable planetary boundary layer, and it was used very successfully for the case of Rayleigh convection by Fitzjarrald [17]. If we substitute into equation (8) Globe and Dropkin's empirical equation $Nu = 0.069Ra^{1/3}$ which covers a range $10^5 < Ra < 10^9$ [12], we have

$$u'/(Pr^{1/3}\kappa/D) = 0.41Ra^{4/9}. \tag{9}$$

Kraichnan [2] obtained the same form as this except for the numerical constant in his theory assuming the mixing-length. It is natural that equations (4) and (9) are almost in agreement with each other.

CONCLUSION

From an experiment for a water layer in a range of the Rayleigh numbers between 3×10^7 and 4×10^9, the following physical pictures of the phenomena in the turbulent thermal convection have been established.

Heat-transfer resistance is concentrated in thermal boundary layers near both surfaces, whose thickness is about two orders of magnitude smaller than the distance between the top and bottom plates. The thermal boundary layer has its own physical mechanism almost independent of central turbulent region, and releases buoyant masses intermittently. A buoyant mass gradually dissipates its heat while it travels away from the surface, and disappears at about twenty times the boundary layer thickness from the surface.

The fluid layer is completely in a turbulent state at high Rayleigh numbers, but large convective motions having a size comparable to the distance between the plates prevail in the central region of the fluid layer, though they are neither geometrically regular nor stationary. Taylor microscale of the turbulent motions is one order of magnitude smaller than the distance between the plates. Conspicuous small eddies having a size of Taylor microscale mix up the warm or cold elements discharged from the lower or upper thermal boundary layer with ambient fluid. Bodies of slightly warm or cold fluid so mixed up in the neighborhood of the either surface are carried by the large convective motions. They come together at a certain position and go up or down with a rush near to the opposite surface, thereby providing the large convective motions with an amount of kinetic energy corresponding to the release of their potential energy. An assumption that the energy-containing eddies have a size comparable to the distance between the plate is further supported by the energy balance between the buoyant production and the viscous dissipation.

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CONVECTION NATURELLE TURBULENTE DANS UNE COUCHE HORIZONTALE D'EAU CHAUFFE PAR LE BAS

Résumé—On présente les résultats d'une étude expérimentale sur le transfert de chaleur dans une couche d'eau entre deux plans horizontaux, chauffée par le bas et refroidie par le haut. L'expérience couvre un domaine de nombre de Rayleigh allant jusqu'à 4×10^9. La résistance thermique est virtuellement limitée par les deux couches limites thermiques proches des plaques. Les couches limites sont exemptes de panaches flottants du fait de leur mécanisme autonome. La région centrale de la couche fluide est presque isotherme. A partir des observations et aussi des considérations théoriques, on conclut qu'il existe des mouvements convectifs ayant une échelle comparable à la distance entre les deux plaques et qu'ils contribuent fondamentalement au transfert thermique dans la région centrale.

TURBULENTE NATÜRLICHE KONVEKTION IN EINER VON UNTEN BEHEIZTEN HORIZONTALEN WASSERSCHICHT

Zusammenfassung—In diesem Aufsatz werden Ergebnisse einer experimentellen Untersuchung zur Wärmeübertragung in einer von zwei horizontalen Platten begrenzten Wasserschicht, die von unten beheizt und von oben gekühlt wird, mitgeteilt. Mit den Experimenten wird ein Bereich der Rayleigh-Zahlen bis zu 4×10^9 abgedeckt. Der Wärmeübertragungswiderstand wird praktisch durch die thermischen Grenzschichten an den heizenden und kühlenden Platten bestimmt. Von der Grenzschicht lösen sich entsprechend ihrem eigengesetzlichen Mechanismus aufwärtsströmende Schlieren ab. Der Kernbereich der Flüssigkeitsschicht ist immer isotherm. Aus optischen Beobachtungen und auch theoretischen Betrachtungen wurde gefolgert, daß konvektive Bewegungen existieren, die ein zum Abstand zwischen den zwei Platten vergleichbares charakteristisches Maß besitzen und die im Kernbereich einen dominierenden Beitrag zur Wärmeübertragung leisten.

ТУРБУЛЕНТНАЯ ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ В НАГРЕВАЕМОМ СНИЗУ ГОРИЗОНТАЛЬНОМ СЛОЕ ВОДЫ

Аннотация — Представлены результаты экспериментального исследования теплообмена в нагреваемом снизу и охлаждаемом сверху слое воды. Опыты проводились при значениях числа Релея до 4×10^9. Сопротивление переносу тепла фактически определяется тепловыми пограничными слоями у нагреваемой и охлаждаемой пластин. В пограничном слое наблюдаются отдельные восходящие струи. Центральная область слоя жидкости является почти изотермической. На основе визуальных наблюдений и теоретических расчетов сделан вывод о том, что масштаб возникающих структур сравним с расстоянием между пластинами и что эти потоки в основном определяют перенос тепла в центре слоя.